

# Investigating inverse methods for determining the dynamic demand

Céline Parzani<sup>1</sup>, Julien Waeytens<sup>2</sup>, Ludovic Leclercq<sup>1</sup>, Rachida Chakir<sup>1</sup>, Gaëlle Jousse<sup>1</sup>

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## ABSTRACT

A key requirement for traffic simulation is the OD matrix that describes the traffic demand i.e. the number of vehicles that want to drive from entries to the different exits. This information is hardly measurable and we often only get indirect measurements, i.e. flow counts within the network. Such data only characterize the dynamic demand when traffic conditions are fluid otherwise they only provide a lower bound. Moreover, usual OD matrix estimation methods use statistical tools (maximum likelihood, entropy maximisation) that disregard traffic dynamics between entries and sensors.

The aim of this paper is to explore an original approach based on inverse method techniques to reconstruct boundary conditions and improve traffic dynamics description. These techniques have notably been performed in the context of water pipe networks for the assessment of water quality for instance (see [1] for details). The main idea of this paper is to adapt the process to traffic flow modelling, considering first order model. Inverse methods have already been tested in traffic flow context but for others applications e.g. flow control at nodes (see [2]) or data assimilation and second order model (see [3]).

The optimal control theory usually uses the boundary conditions and the initial data as control functions. A cost function depending on initial and boundary conditions is introduced to measure the distance between observed and simulated data. In this study, traffic flow dynamics is described by the LWR model (see [3]), which is based on a hyperbolic partial differential equation of first order, which describes the

conservation of vehicles in time and space:  $\frac{\partial k}{\partial t} + \frac{\partial Q}{\partial x} = 0$ , with  $k$  the vehicle density and  $Q$  the vehicle

flow over time  $t$  and space  $x$ . The model is completed with a fundamental diagram describing the relation between the flow and the density. Based on the method developed in [1] and [4], the boundary conditions are considered as boundary controls. The reconstruction process then consists in defining fictitious boundaries and identifying the optimal densities on these boundaries that minimize the data misfit. In order to minimize the cost function, a classical descent method is usually employed. One of the key points is the evaluation of the gradient of the cost functions that appear in the problem. This is done with the help of the adjoint system using the continuous conservation law equation. Adjoint systems for conservation laws have also been derived in [6-8]. The main steps of the descent algorithm can be summarized as follows:

- solve the direct problem with the boundary control from the previous step,
- solve the adjoint problem,
- compute the descent direction and the descent step,
- update the boundary control.

In this study, we will also investigate a second method to solve directly the optimisation problem using efficient algorithms such as genetic ones. The two approaches will then be compared.

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<sup>1</sup>Laboratoire Ingénierie Circulation Transport LICIT  
IFSTTAR / ENTPE – Université de Lyon  
Rue Maurice Audin, F-69518 Vaulx-en-Velin

<sup>2</sup>Laboratoire Instrumentation, Simulation et Informatique Scientifique  
Université Paris-Est, IFSTTAR  
boulevard Newton, F-77747 Champs sur Marne, France

In order to validate the free-flow process, this methodology is first applied on a junction with one ingoing and two outgoing roads for free traffic. Then, it will be extended to more complex network configurations including uncertainties due to route choices.

A second point that will be addressed with this methodology is the number of detectors required for applying inverse methods with route choices.

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